

Probing the electron charge distribution via Kapitza-Dirac diffraction

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Abstract

We analyze the diffraction of elementary systems as the electron by light gratings when they are described by charge distributions instead of the usual point-like form. The treatment of the problem is based on the introduction, in analogy with atomic polarizability, of state-dependent non-permanent multi-pole moments for the charge. The diffraction patterns can provide bounds on these moments. With this approach we can experimentally explore some aspects of the interpretation of the wave picture of single charges.

1 Introduction

The question of the elementary character of seemingly structureless particles as the electron frequently emerges in the physical literature. These analysis mainly focus on three interconnected aspects: the existence of substructures [1, 2], the presence of dipole or higher moments [3, 4], and the actual size of the electron (as opposed to a point-like object) [5, 6]. These works are based on a large variety of experimental techniques such as Thomson and Compton scattering, Penning traps, high energy electron-positron collisions or precision measurements of the energy states of YbF molecules.

We consider in this paper a particular aspect of the problem, closely related to the wave-particle duality. Quantum systems can behave as particles (point-like) or waves (spatially extended). The typical examples of the last case are diffraction experiments. We can ask for the behavior of a charge in this framework: does it also show spatially extended characteristics? Equivalently, does it possess multi-pole moments different from zero in these circumstances? We address some aspects of the question by invoking techniques of optical (Kapitza-Dirac effect [7, 8]) and atomic (non-permanent moments [9]) physics.

Comparing the diffraction patterns of point-like and spatially extended charges we can study the differences between the two types of charge distributions. In

order to carry out the comparison one must derive the expected patterns in both cases. The laser interaction with a point-like charge is given by the ponderomotive potential [10]. We must evaluate the laser interaction with a charge distribution. When the characteristic length of the charge distribution is smaller than the wavelength of the driving laser we can approach the problem via a multipole expansion of the distribution. Using this expansion it is simple to derive the diffraction patterns for this case in the approximation of initial plane wave states. This simple but illustrative example shows that the patterns differ in both cases. Determining these detection probabilities we can introduce bounds on the values of the multipole moments. The advantage of the Kapitza-Dirac effect is that the experiment with electrons has been carried out in [11]. Minor modifications of the arrangement, introduced to use adequate laser wavelengths, would allow us to confront theoretical results with actual data.

From the beginning it must be stressed that the moments we study in the paper would not be intrinsic or permanent to the electron. Permanent moments would be present under any circumstances. There have been experiments with YbF molecules providing precision bounds on their possible values [4]. In contrast, we are interested into moments that would only manifest when the charge is forced to behave as an extended object, that is, non-permanent moments. There is a clear analogy of this proposal with atomic polarizability, where an external field can induce non-intrinsic moments in the atom [9].

2 Ponderomotive potential for distributions

Our first task is to determine the form of the light-matter interaction for charge distributions. We must know the form of the interaction potential to be introduced into the Schrödinger equation. As in the point-like case we assume that this potential has the classical form. Then we must derive the form of the interaction between the charge distribution and the light field in the classical electromagnetic formalism. We expect, by similitude with the point-like case, that the Schrödinger equation with the interaction potential of classical charge distributions will correctly describe the quantum behavior of extended charges. In this section, from now on, all our considerations will be from a classical nature. In particular, the charge density associated with the elementary charge is classical.

The interaction of a point-like electron with a laser is described by the ponderomotive force,

$$\mathbf{F}_P = -\frac{e^2}{4m\omega_L^2}\nabla\mathbf{E}^2(\mathbf{r}) \quad (1)$$

which can be expressed in terms of a potential, $\mathbf{F}_P = -e\nabla V_P$, denoted as the ponderomotive potential:

$$V_P(\mathbf{r}) = \frac{e}{4m\omega_L^2}\mathbf{E}^2(\mathbf{r}) \quad (2)$$

with \mathbf{E} the electric field of the laser and ω_L its frequency (the temporal dependence is averaged over the laser period). The energy of the charge in this potential is eV_P .

In the particular case of a standing light wave with spatial and temporal dependence $\cos k_L x \cos \omega_L t$, with k_L the light wavelength and x the coordinate in the laser propagation direction, it can be written as

$$V_P(x) = \frac{e\mathbf{E}_0^2}{4m\omega_L^2} \cos^2 k_L x \quad (3)$$

We consider now how an electron behaves in that potential when we assume that the charge is a distribution instead of a point-like object. The energy of a charge density distribution, $\rho(\mathbf{r})$, in a ponderomotive potential is

$$U_P = \int \rho(\mathbf{r}) V_P(\mathbf{r}) d^3 \mathbf{r} \quad (4)$$

This expression gives the total energy of the charge in the ponderomotive potential. There are two regimes. The first one refers to situations where the relation between the typical scales of the charge distribution and the wavelength of the laser allows for a multi-pole approach to the problem. When this is not possible, the second regime, we must resort to other techniques. In this paper we shall restrict our considerations to the first case.

When the potential varies smoothly in the region where $\rho(\mathbf{r})$ is defined we can expand V_P around a point taken as the origin [12]

$$V_P(\mathbf{r}) = V_P(0) + \mathbf{r} \cdot (\nabla V_P)(0) + \frac{1}{2} \sum_{i,j} r_i r_j \frac{\partial^2 V_P}{\partial r_i \partial r_j}(0) + \dots \quad (5)$$

Introducing this expression into the energy equation and using the relation $\int \rho(\mathbf{r}) d^3 \mathbf{r} = e$, we have

$$U_P = eV_P(0) + \mathbf{D} \cdot (\nabla V_P)(0) + \frac{1}{2} \sum_{i,j} Q_{ij} \frac{\partial^2 V_P}{\partial r_i \partial r_j}(0) + \dots \quad (6)$$

with the usual dipole $\mathbf{D} = \int \mathbf{r} \rho(\mathbf{r}) d^3 \mathbf{r}$ and quadrupole moments $Q_{ij} = \int r_i r_j \rho(\mathbf{r}) d^3 \mathbf{r}$. It is important to remark that in our problem the moments do not refer to a set of charges but to a single charge with a spatial distribution.

In order to the multi-mode expansion be useful we expect the contribution of the terms to decrease when its order increases. When this is so, only the zero, dipole and quadrupole moments are relevant and we can neglect the rest. We analyze this point later.

3 Diffraction patterns for distributions

Once derived the interaction potential for point-like and extended charges we can study the quantum diffraction patterns in both cases. We consider a situation

that can be solved analytically, that where the initial state of the electron can be described by the approximation of a plane wave. Of course, to use plane waves is an oversimplification in realistic problems. However, it provides a simple example that illustrates the differences between both approaches.

In this approximation the solution for a point-like charge is well-known [8]. The problem can be taken as an one-dimensional one, where we only must care about the transversal variables, that is, those in the direction of propagation of the light. The diffractive regime is reached when the potential is much larger than the recoil shift ($\epsilon = \hbar^2 k_L^2 / 2m$). This condition is equivalent to neglect the free term of the Hamiltonian when compared to the interaction term (Raman-Nath approximation) [8]. In this regime the initial plane wave state of the electrons, $\psi(0) = e^{ik_0 x}$ with k_0 the initial wavelength, evolves as

$$\psi(t) = e^{ieV_P(x)t/\hbar} \psi(0) = e^{ieV_0 t/2\hbar} \sum_{n=-\infty}^{\infty} i^n J_n \left(\frac{eV_0 t}{2\hbar} \right) e^{i(2nk_L + k_0)x} \quad (7)$$

where we have introduced the notation $V_0 = e\mathbf{E}^2/4m\omega_L^2$ and we have used the expression $e^{i\xi \cos \varphi} = \sum_{n=-\infty}^{\infty} i^n J_n(\xi) \exp(in\varphi)$, with J_n the n -th order Bessel's function. The wavelength of the electron changes by even multiples of k_L . The probability of detecting the particle in the n -th diffraction order is given by $|J_n|^2$.

We move now to the case of charge distributions. The energy U_P in a standing wave reads

$$U_P(x) = eV_0 \cos^2 k_L x - V_0 D k_L \sin 2k_L x - V_0 Q k_L^2 \cos 2k_L x + \dots \quad (8)$$

If we assume, as in the standard case, that $U_P \gg \epsilon$ the evolution of the initial state is

$$\begin{aligned} \psi(t) = e^{iU_P(x)t/\hbar} \psi(0) = e^{ieV_0 t/2\hbar} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} i^n \times \\ J_n \left(\frac{V_0 t}{\hbar} \left(\frac{e}{2} - Q k_L^2 \right) \right) J_m \left(-\frac{V_0 D k_L t}{\hbar} \right) e^{i(2(n+m)k_L + k_0)x} \end{aligned} \quad (9)$$

In the derivation we have used the relation $e^{i\xi \sin \varphi} = \sum_{n=-\infty}^{\infty} J_n(\xi) \exp(in\varphi)$.

The final state is a sum over all the states whose wavelength differs from k_0 by even multiples of k_L . Both expressions, for point-like and extended charges, are sums in a basis of plane waves but with different coefficients. Then both diffraction patterns are different. The simplest form to quantify these differences is to determine the weights of each mode. This is equivalent to evaluate the probability of diffraction with a given value of momentum change. For instance, for the zero order (no diffraction) in the first case we have a probability of detection $|J_0(eV_0 t/2\hbar)|^2$, whereas for a distribution charge it is $|J_0(V_0 t(e/2 - k_L^2 Q)/\hbar) J_0(V_0 k_L D t/\hbar)|^2$. Both expressions are clearly different. Varying the

free parameters of the problem V_0 (intensity of the laser) and t (interaction time between laser and electron), we could fit the last equation to the experimental data and to determine D and Q .

As signaled before, the approach followed here only makes sense when the lower terms are the relevant ones. This condition can be easily obtained with the above expressions. As V_0 is common to all the orders, and the trigonometric functions are bound (their absolute values are equal or smaller than one) the relative intensity of the terms is given by $1, Dk_L/e, Qk_L^2/e, \dots Q_mk_L^m/e, \dots$. Then we must have $Dk_L < e$, $Qk_L^2 < e, \dots$ and $Q_m < Q_n k_L^{n-m}$, $m > n$.

In actual experiments, such as that reported in [11], the plane wave approximation is clearly an oversimplification. In order to compare the expected results with those of realistic experiments we must carry out a numerical simulation of the Schrödinger equation for the arrangement with the interaction potential of Eq. (8) and the actual state in the experiment. Varying D and Q and comparing with the experimental results we could discard ranges of values incompatible with the experimental data.

We must also evaluate the order of magnitude of the characteristic lengths that can be actually explored with the multi-pole expansion. We denote by \mathcal{L} the characteristic length. The standard condition to observe an object with light is $\mathcal{L} \approx \lambda_L$, that is, the laser wavelength must be of the order of the characteristic length. At present, the shorter laser wavelengths for which the Kapitza-Dirac effect seems to be experimentally accessible lie on the X-ray domain [13]. Beyond that domain, in the actual state of the art, there are not coherent light sources of enough intensity. Thus, with present day technology we could explore via Kapitza-Dirac diffraction multi-pole structures of the order of $\mathcal{L} \approx 10^{-10}m$.

Finally, we want to remark again on the concept of non-permanent moments. A charge distribution can deviate from the point-like form in two different ways. The charge can have an intrinsic structure independently of its behavior or state. We say that it is a permanent contribution and we can speak of permanent moments. On the other hand, if the charge distribution can change with the state the electron it can acquire additional moments. They are dependent on the state of the system and are denoted non-permanent moments. The situation resembles that in atomic physics, where the moments of an atom are different when it is placed in an external electric field due to polarizability effects [9]. In both cases we need an external process forcing the electron (diffraction grating) or the atom (electric field) to a behavior where the non-permanent contributions manifest. In [13] non-permanent atomic moments are briefly discussed in the context of Kapitza-Dirac diffraction. The moments used here are non-permanent, they are only present when the electron diffracts.

4 Extended properties and the wave picture

In the second part of the paper we suggest that the above scheme can be used to study an interpretational question, that related to the physical meaning of the wave picture. In diffraction experiments we associate a wave picture with the system. Does this wave picture describe objective extended physical properties or it is only related to the statistical nature of the wave function reflected in the possibility of detecting the electron at different places after the diffraction? Clearly, one of these objective properties would be the existence of (non-permanent) multi-pole moments different from zero.

The natural framework for this discussion is the wave-particle duality, where one associates exclusive wave or particle pictures with the system. In the standard complementarity formalism the wave picture is interpreted in a statistical sense. We do not deal with a physical wave. The spatially extended properties of the system, for instance the detection of the particle at different locations in different repetitions of the experiment, only correspond to a statistical feature of the mathematical description. In contrast, other authors have suggested that a more physical explanation is possible. In this alternative approach the system would possess objective (and, in principle, testable) extended physical properties.

In our case these properties should be associated with the charge of the electron, the relevant element in the light-matter interaction. First of all, it is evident that if we associate a wave picture with the electron during the diffraction process then, because of the unicity of the system, we must also use a wave picture for the charge. In a physical (non-statistical) interpretation the charge would be an extended object and, in consequence, it could be described invoking a multi-pole formalism. These moments would not be present in the particle picture and, consequently, must be considered from a non-permanent nature. In contrast, in the statistical interpretation the charge does not have extended properties, which only manifest in the detection at different places of the point-like electron.

From a more technical point of view, if the charge is an extended object its wave function must be calculated from an evolution equation taking into account this extended aspect. As discussed in Sect. 2 we must use the Schrödinger equation with the U_P potential. The solution of the equation will include the contributions of the hypothetical non-permanent multi-pole moments. In contrast, in the statistical framework one uses the evolution equation with the point-like potential.

We summarize the above considerations. The diffraction patterns provide an experimental method to discriminate between the physical and statistical interpretations of the wave picture. If one can really ascribe objective extended physical properties to the wave, it is natural to identify them with the existence of non-permanent moments different from zero, which lead to testable deviations from the statistical interpretation. If no definitive deviation is obtained in the experiments, these tests will at least provide bounds on the values

of the non-permanent moments, empirically constraining physically objective interpretations.

5 Discussion

We have studied Kapitza-Dirac diffraction of electrons when the charge is described by a distribution instead of the usual point-like idealization. Our fundamental technical assumption is that the evolution of the system is ruled by the Schrödinger equation, but with the laser-electron interaction potential given by U_P , the generalization of the ponderomotive potential to charge distributions. With this assumption it is simple to derive the diffraction patterns when the size of the distribution is small in comparison with the laser wavelength. Here, we have only considered the oversimplified case of electrons initially in plane wave states. For more realistic initial electron states one must resort to numerical simulations. By comparison to repetitions of the experiment [11] with adequate laser wavelengths and electron states, we could infer bounds on the values of the multi-pole moments. When the conditions for a multi-pole expansion do not hold we should consider other types of techniques.

Our proposal has two possible applications. On the one hand, it provides a new method to study deviations from the point-like form in elementary systems. On the other hand, it introduces a novel tool to analyze some aspects of the interpretation of the wave picture.

With respect to the first point we must compare our proposal with other approaches raising the same question. As signaled in the Introduction very stringent tests of the electron size have been conducted using Penning traps [5, 6]. In these experiments the magnetic moment of the electron is determined with very high accuracy, agreeing extremely well with the values predicted by Quantum Electrodynamics. Introducing models of finite size electrons the authors derived upper limits for the electron size of respectively $10^{-20}m$ and $10^{-22}m$. Our method provides an alternative and independent way to estimate limits on the effective electron size. In a related context the permanent dipole moment of the electron has been analyzed both theoretically and experimentally. In the Standard Model a tiny electric dipole moment, $d_e < e \times 10^{-40}m$, is associated with the electron. This value, although extremely small, could be used to distinguish between the standard theory and some proposed extensions [3]. Recent, high accuracy measurements provide an upper limit for d_e of $e \times 10.5 \times 10^{-30}m$ [4]. The dipole moments considered in these experiment are from the permanent type because the electrons in Penning traps or YbF molecules are not forced to a diffraction-like behavior. In addition, it must be noted that the measurements in [4] are carried out in electrons in bound states. One must consider the possibility that dipole moments can differ for bound and non-bound states. Clearly, the values obtained in [4] or predicted by the Standard Model would provide a completely negligible contribution of the dipole term to the modifications of

the diffraction pattern. All the hypothetical effects that could be observed in a diffraction experiment should be attributed to non-permanent contributions.

In relation to the second point our proposal could be on the basis of a test discriminating between statistical and physically objective interpretations of the wave aspect of quantum systems. At least, it can provide quantitative bounds on the viability on the second type of interpretation. The non-permanent moments play a fundamental role in this discussion. The polarizability effects in atomic physics suggest an interesting analogy with these moments. Note, however, that this analogy is only partial because in the atomic case there is an underlying charge structure that is not present in the electron. The non-permanent moments of the electron would be state-dependent and could not be associated with any substructure.

To end the paper we want signal some similarities between the ideas here presented and the work [14], where the radiative properties of electrons interacting with a laser were studied as a function of its charge distribution. These authors concluded that during the emission process the electron cannot be treated as an extended charge but as a point-like emitter, even when the spread of the electron wavepacket is comparable to the wavelength of the driving laser. The resemblances to our approach are evident. However, there is a fundamental difference between them, in [14] the electron is not explicitly forced to behave in a wave-like way. Moreover, diffraction is a smooth process whereas emission corresponds to a sharp evolution.

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